
Time Evidence Fusion Network: Multi-source View in Long-Term Time Series Forecasting

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Abstract

In real-world scenarios, time series forecasting often demands timeliness, making research on model backbones a perennially hot topic. To meet these performance demands, we propose a novel backbone from the perspective of information fusion. Introducing the Basic Probability Assignment (BPA) Module and the Time Evidence Fusion Network (TEFN), based on evidence theory, allows us to achieve superior performance. On the other hand, the perspective of multi-source information fusion effectively improves the accuracy of forecasting. Due to the fact that BPA is generated by fuzzy theory, TEFN also has considerable interpretability. In real data experiments, the TEFN partially achieved state-of-the-art, with low errors comparable to PatchTST, and operating efficiency surpass performance models such as Dlinear. Meanwhile, TEFN has high robustness and small error fluctuations in the random hyperparameter selection. TEFN is not a model that achieves the ultimate in single aspect, but a model that balances performance, accuracy, stability, and interpretability. Code is available at this repository: <https://github.com/ztxtech/Time-Evidence-Fusion-Network>.

1 Introduction

The changes of things are accompanied by the passage of time, and time series appear in more and more application scenarios, such as health diagnosis, financial analysis, weather forecasting, and so on. Optimizing structure of backbone is an effective way to improve the performance of time series forecasting, where linear-based models such as Dlinear often outperform transformer-based models such as Crossformer [18, 20]. Therefore, we hope to propose a model that balances performance, prediction accuracy, stability, and interpretability to provide support for highly reliable scenarios with time requirements.

Evidence theory is a model of uncertain reasoning commonly used for decision-making in the fusion of multiple information [14, 6]. Meanwhile, evidence theory is often appeared in reasoning framework based on symbolic logic, with high interpretability. Denoeux has implemented evidence

neural networks for classification tasks [8]. Therefore, we propose the Time Evidence Fusion Network (TEFN) for regression tasks. The strategy adopted by TEFN is to divide the time series into two perspectives: time and channels, treating each channel/time as an independent information source, and using an information fusion framework for prediction in Figure 1. Therefore, TEFN is a simple and naive model, without any technical modifications, and is a new backbone.

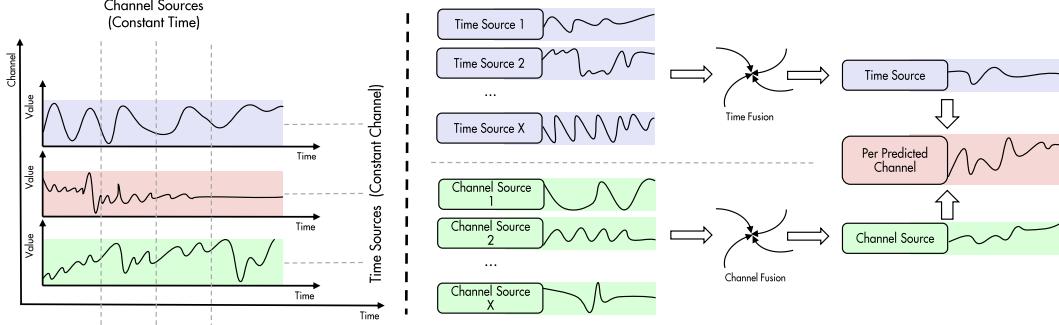


Figure 1: The channel dimension and time dimension of multivariate time series can be considered as different sources of information. Firstly, merge information sources of the same type, and finally integrate information sources from both channel and time dimensions to complete the final prediction.

In TEFN, we propose a novel module Basic Probability Assignment (BPA) Module. Based on the evidence theory of decision-making process, BPA maps different sources of information to distributions related to the target outcome. The target distribution is achieved by the fuzzy membership of different fuzzy sets [7]. In image processing, convolution condenses the required features of the target [4]. Unlike convolution, BPA is an expansion process in Figure 2. On the contrary, BPA considers all possibilities in the event space composed of subsets of the sample space, expanding on different events, which is more effective for simple structures like time series and can fully explore the different hidden information inside. For different distributions generated by multiple information sources, we consider sources as samples and use the expected forecasting values as the final result.

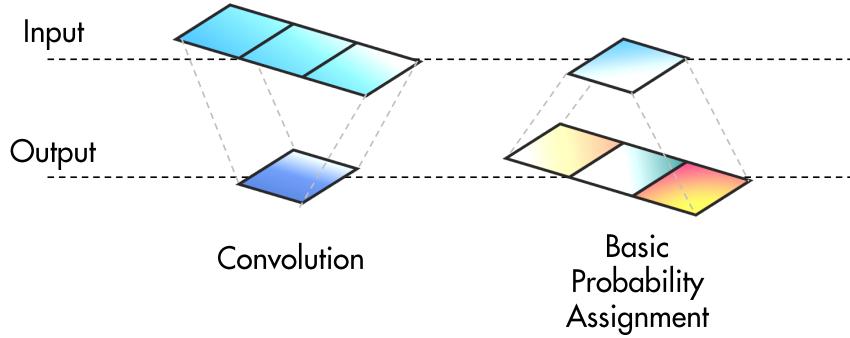


Figure 2: Comparison between convolution and basic probability assignment: convolution condenses information while basic probability diverges to consider different possibilities.

In this article, we focus on the practical application details of TEFN, including its accuracy, memory usage, training speed, parameter sensitivity, and interpretability. At the same time, in order to ensure fairness, the experimental data and configuration files for comparison are all from open-source projects. TEFN achieves state-of-the-art (SOTA) on multiple real datasets, while training time and model parameters are much smaller than models based on Transformer [20] or Patch [12]. For random hyperparameter selection, TEFN exhibits weak fluctuations and is a stable model. The contributions of this article are as follows:

- This article proposes a new evidential backbone for extracting potential distributions of simple data structures.

- A neural network TEFN based on BPA for time series forecasting has been proposed, which leads previous research in accuracy, efficiency, stability, and interpretability.
- This article explicitly introduces relevant concepts from evidence theory, such as mass function, into the structure of neural networks, providing a new implementation approach for uncertainty reasoning.

2 Related Work

2.1 Dempster-Shafer Theory

Dempster-Shafer theory, also known as Evidence Theory, operates on a different framework compared to traditional probability theory [14, 6]. It relies on the concept of mass functions, which provide a more flexible representation of uncertainty, accommodating weaker constraints than traditional probabilities. When combining two mass functions, denoted as m_1 and m_2 , the resulting fused mass function m is determined using the Dempster-Shafer Rule (DSR), as described in Equation 1.

$$m(A) = \sum_{B \cap C = A} m_1(B) * m_2(C) \quad (1)$$

2.2 Evidence Decision Making

The process of Evidence Decision Making can be delineated into several sequential steps: basic probability assignment (BPA), evidence fusion, and decision making (DM) [7]. BPA serves as a pivotal initial phase, constituting a mapping from raw data to mass distribution, akin to the path from raw data to probability distribution. Subsequently, evidence fusion integrates distinct mass distributions from various data channels, culminating in a rationalized mass distribution. Finally, decision making transpires as the progression from the attained distribution towards the designated target.

3 Time Evidence Fusion Network

The structure of TEFN is naive, and well performance can be achieved through a structure with only 2 BPA module similar to only 2 convolution kernels. We believe that **the new architecture of TEFN is more like a pure backbone model**. The overall structure of TEFN is in Figure 3.

3.1 Time Normalization and De-normalization

In order to accelerate converging to local optima more quickly, we referred to the normalization methods of Stationary in TEFN [11]. The normalization process calculates the mean μ in Equation 2 and variance difference σ^2 in Equation 3 for each piece of the time series x . The normalization and de-normalization processes are a pair of symmetric operations in Equation 4 and Equation 5, where $Net(\cdot)$ is the neural network operation performed on the normalized time series x_{Norm} .

$$\mu = \frac{1}{|x|} \sum_{x_i \in x} x_i \quad (2)$$

$$\sigma^2 = \frac{1}{|x|} \sum_{x_i \in x} (x_i - \mu)^2 = \frac{1}{|x|} \sum_{x_i \in x} x_i^2 - \mu^2 \quad (3)$$

$$x_{Norm} = \frac{x - \mu}{\sigma} \quad (4)$$

$$x_{De-Norm} = \sigma * \hat{y}_{Norm} + \mu = \sigma * Net(x_{Norm}) + \mu \quad (5)$$

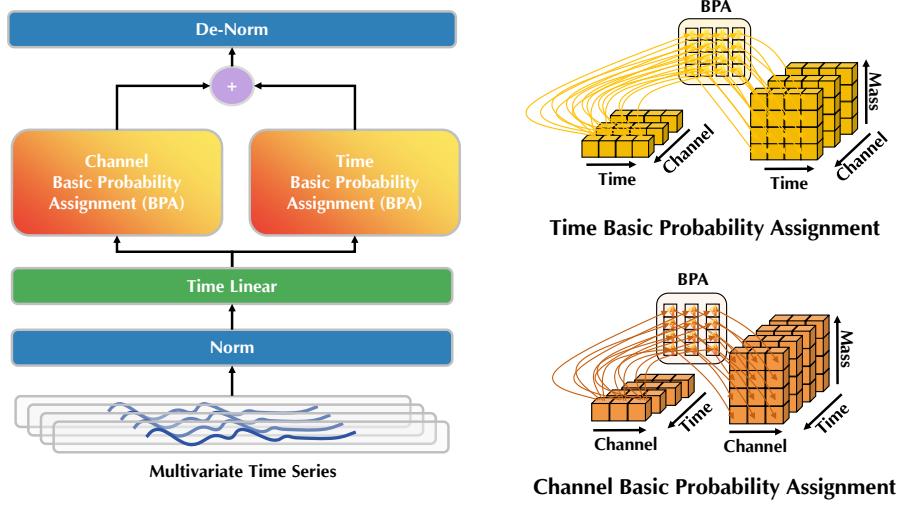


Figure 3: The overall structure of TEFN

3.2 Time Dimension Projection

In TEFN, a linear layer is used to project the length L_{in} of the time dimension to length $L_{in} + L_{pred}$ where L_{pred} is prediction length of current target.

3.3 Basic Probability Assignment Module

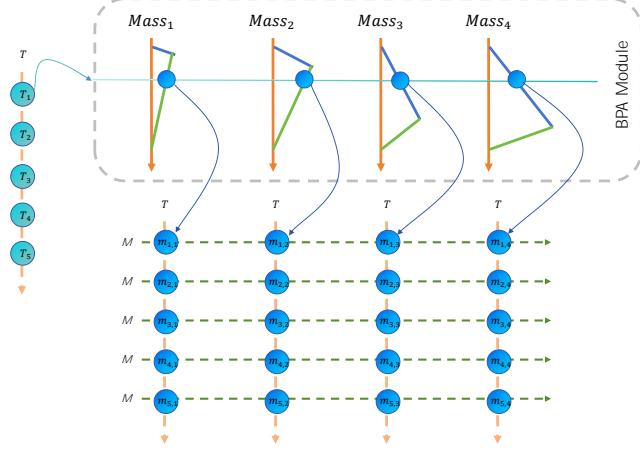


Figure 4: The operation process of a BPA module for a time series T of length $L = 5$. The sample space size in the BPA module is set as $|S| = 2$, and the event space size is set as $|2^S| = 2^{|S|} = 4$. The operation of the BPA module is similar to that of the convolution module, but the BPA module diffuses different masses instead of summing them in the convolution.

According to Evidence Theory, each dimension of a time series can be reduced to a mass function in the same set of event spaces. The event space of evidence theory usually chooses power sets 2^S of sample space S . If it is a classification problem, the sample space is all the labels. For regression problems, the sample space needs to be treated as anonymous samples, where the samples are mutually exclusive. Similarly, whether it is a classification or regression problem, each dimension of

the data will generate a mass distribution in the event space. Fuzzy numbers commonly used in the generation of mass distribution of evidence.

Here we use a parameterized fuzzy membership μ in Equation 6, where D (time dimension T and channel dimension C are available) is the dimension that generates the mass function, j is corresponding to dimension D and i is corresponding to the other one. Parameter w and b represent the slope and intercept of the membership function, and k is corresponding to event dimension F .

$$m_{D,i,j,k} = \mu(x_{Norm,i,j}) = w_{D,j,k} * x_{Norm,i,j} + b_{D,j,k} \quad (6)$$

3.4 Expectation Fusion

For multivariate time series in shape $|T| * |C|$, there are two dimensions: time dimension T and channel dimension C . So, the different mass distributions m_C, m_T can be generated simultaneously. And the fusion method here uses expectations in Equation 7 where $y_{j,k}$ is parameter and $m'_{i,j,k}$ is corresponding mass function. Summing part $y_{j,k} * m'_{i,j,k}$ can be regarded as a linear transform of $m_{i,j,k}$, and expectation is calculated by summing $m_{i,j,k}$ in Equation 7. There are two reasons why DSR are not used:

1. The mathematical operation corresponding to DSR is the point-to-point multiplication of mass distributions within the same dimension, which has a high computational complexity. At the same time, if the dimension is too high, it will lead to the disappearance of accuracy within a finite precision.
2. DSR is sensitive to one extreme distribution, such as a distribution with only one element being 1, which can result in the final fusion result of the entire mass being directly equal to that distribution.

$$\hat{y}_{Norm,D,i,j} = E_{D,F}(y_{j,k}) = \sum_{\substack{j \in [1, |D|] \\ k \in [1, |2^S|]}} y_{j,k} * m'_{i,j,k} = \sum_{\substack{j \in [1, |D|] \\ k \in [1, |2^S|]}} m_{i,j,k} \quad (7)$$

At the end, results of time dimension T and channel dimension C will aggregate in Equation 8 where \hat{y}_{norm} is normalized predicted results. The final prediction results are restored through the de-normalization layer.

$$\hat{y}_{norm} = \sum_{D \in \{T, C\}} \hat{y}_{Norm,D} = \hat{y}_{Norm,T} + \hat{y}_{Norm,C} \quad (8)$$

4 Experiments

Dataset We will use TEFN for long-term prediction tasks. Table 1 shows the basic settings used in the experiment, including the datasets. In terms of datasets, we have selected 5 common time series datasets, including Electricity [1], ETT (4 subsets) [10], Exchange [17], Traffic [2], Weather [3]. More dataset features are in the Appendix Table 5.

Table 1: Datasets and Basic Settings

| Tasks | Datasets | Metrics | Series Length |
|-----------------------|--|----------|---------------|
| Long-term Forecasting | ETT (4 subsets), Electricity, Traffic, Weather, Exchange | MSE, MAE | 96~720 |

Baseline The comparative experimental models include PatchTST (ICLR 2023) [12], Crossformer (ICLR 2023) [20], TiDE [5], TimeNet (ICLR 2023) [16], ETSformer [15], LightTS [19], Dlinear (AAAI 2023) [18], FEDformer (ICML 2022) [21], Stationary (NeurIPS 2022) [11] and Autoformer (NeurIPS 2021) [17]. The configurations of the comparative experiments are from the Time Series Library (<https://github.com/thuml/Time-Series-Library>). More implementation details are in the Appendix Section A.

4.1 Forecasting Result

TEFN partially achieved SOTA performance in long-term forecasting target in Table 2. It is worth noting that in the case of a simple structure with tiny parameters in Section 4.4, achieved approximate or even better performance than previous SOTA models with larger parameters, such as PatchTST [12]. Full forecasting results are in the Appendix Table 7.

Table 2: Prediction error. All results are the average of four predicted lengths {96, 192, 336, 720}.

| Models | TEFN (Ours) | | PatchTST [12] | | Crossformer [20] | | TIDE [5] | | TimesNet [16] | | ETSformer [15] | | LightTS [19] | | Dlinear [18] | | FEDformer [21] | | Stationary [11] | | Autoformer [17] | |
|-----------------------|----------------|--------------|------------------|--------------|---------------------|--------------|--------------|--------------|------------------|--------------|-------------------|-------|-----------------|-------|-----------------|--------------|-------------------|-------|--------------------|--------------|--------------------|-------|
| | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE |
| ETTm1 | 0.403 | 0.398 | 0.387 | 0.400 | 0.513 | 0.496 | 0.419 | 0.419 | 0.400 | 0.406 | 0.429 | 0.425 | 0.435 | 0.437 | 0.403 | 0.407 | 0.448 | 0.452 | 0.481 | 0.456 | 0.588 | 0.517 |
| ETTm2 | 0.286 | 0.327 | 0.281 | 0.326 | 0.757 | 0.610 | 0.358 | 0.404 | 0.291 | 0.333 | 0.293 | 0.342 | 0.409 | 0.436 | 0.350 | 0.401 | 0.305 | 0.349 | 0.300 | 0.347 | 0.327 | 0.371 |
| ETTh1 | 0.441 | 0.429 | 0.469 | 0.454 | 0.529 | 0.522 | 0.541 | 0.507 | 0.458 | 0.450 | 0.542 | 0.510 | 0.491 | 0.479 | 0.456 | 0.452 | 0.440 | 0.460 | 0.570 | 0.537 | 0.496 | 0.487 |
| ETTh2 | 0.380 | 0.402 | 0.387 | 0.407 | 0.942 | 0.684 | 0.611 | 0.550 | 0.414 | 0.427 | 0.439 | 0.452 | 0.602 | 0.543 | 0.559 | 0.515 | 0.437 | 0.449 | 0.526 | 0.516 | 0.450 | 0.459 |
| Electricity | 0.215 | 0.292 | 0.205 | 0.290 | 0.244 | 0.334 | 0.251 | 0.344 | 0.192 | 0.295 | 0.208 | 0.323 | 0.229 | 0.329 | 0.212 | 0.300 | 0.214 | 0.327 | 0.193 | 0.296 | 0.227 | 0.338 |
| Exchange | 0.362 | 0.402 | 0.259 | 0.281 | 0.259 | 0.315 | 0.271 | 0.320 | 0.416 | 0.443 | 0.410 | 0.427 | 0.385 | 0.447 | 0.354 | 0.414 | 0.519 | 0.506 | 0.461 | 0.454 | 0.613 | 0.539 |
| Traffic | 0.623 | 0.372 | 0.367 | 0.404 | 0.940 | 0.707 | 0.370 | 0.413 | 0.620 | 0.336 | 0.621 | 0.396 | 0.622 | 0.392 | 0.625 | 0.383 | 0.610 | 0.376 | 0.624 | 0.340 | 0.628 | 0.379 |
| Weather | 0.260 | 0.283 | 0.481 | 0.304 | 0.550 | 0.304 | 0.760 | 0.473 | 0.259 | 0.287 | 0.271 | 0.334 | 0.261 | 0.312 | 0.265 | 0.317 | 0.309 | 0.360 | 0.288 | 0.314 | 0.338 | 0.382 |
| 1 st Count | 8 | 20 | 16 | 13 | 1 | 0 | 1 | 0 | 7 | 5 | 0 | 1 | 0 | 0 | 1 | 0 | 5 | 0 | 1 | 1 | 0 | 0 |
| 2 nd Count | 13 | 7 | 7 | 15 | 1 | 3 | 4 | 3 | 6 | 5 | 2 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 5 | 4 | 0 | 0 |
| 3 rd Count | 5 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 11 | 20 | 4 | 0 | 3 | 0 | 6 | 4 | 2 | 1 | 0 | 1 | 0 | 1 |

4.2 Hyperparameter Sensitivity

To test the hyperparameter sensitivity of TEFN, we conducted a 2D traversal of the learning rate $lr \in \{10^{-2}, 5 \times 10^{-2}, 10^{-1}\}$ and sample space size $|S| \in \{0, 1, 2, 3, 4, 5, 6\}$. We take the Electricity-96 task as an example. TEFN demonstrates weak sensitivity to parameters, but as the sample space increases, the error magnitude also increases, leading to a certain degree of overfitting in the model as shown in Figure 5. It is worth noting that the order of magnitude of the MSE in Figure 5 is 10^{-4} , while the order of magnitude of the compared MSE is 10^{-3} , so the fluctuation of different hyperparameter in error metrics is small. Full hyperparameter sensitivity experiment results are in the Appendix Table 8.

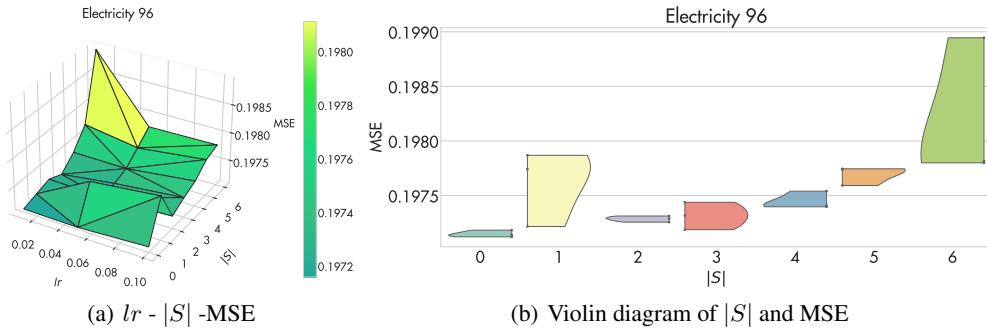


Figure 5: Traversing error of Electricity 96 task: 3D visualization and violin diagram. The points on the number axis of the violin chart reflect the density of learning rate.

Additionally, we conducted statistical analysis on the error variance of different tasks as shown in Table 3. The order of magnitude of variance is 10^{-4} , indicating extremely small fluctuation. Random parameter selection within the traversal range of TEFN yields stable prediction results.

4.3 Ablation Study

TEFN is formed by the fusion of time information sources T and channel information sources C . In the ablation experiment, we made separate predictions for the time and channel parts of TEFN as shown in Table 4. Across most forecasting tasks, the fusion $C + T$ error is lower than that of

Table 3: The variance of model error for traversal learning rate and sample space overhaul

| Metric (Var) | | Dataset | | | | | | | |
|-----------------|-----|-------------|--------|--------|--------|--------|----------|---------|---------|
| | | Electricity | ETTh1 | ETTh2 | ETTm1 | ETTm2 | Exchange | Traffic | Weather |
| MSE | 96 | 0.0000 | 0.0000 | 0.0002 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0001 |
| | 192 | 0.0001 | 0.0000 | 0.0003 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0002 |
| | 336 | 0.0001 | 0.0000 | 0.0002 | 0.0004 | 0.0000 | 0.0023 | 0.0000 | 0.0001 |
| | 720 | 0.0000 | 0.0000 | 0.0004 | 0.0001 | 0.0000 | 0.0038 | 0.0000 | 0.0001 |
| MAE | 96 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0002 |
| | 192 | 0.0001 | 0.0000 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0002 |
| | 336 | 0.0001 | 0.0000 | 0.0001 | 0.0001 | 0.0000 | 0.0009 | 0.0000 | 0.0001 |
| | 720 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0007 | 0.0000 | 0.0000 |

individual time T or channel C error. Although partial use of a single channel may further reduce prediction error, the error reduction is not significant (error of Electricity only varies 0.001). It displays that TEFN effectively filters out time T and channel C information, and individual time T and channel C information alone cannot effectively construct the evolution pattern of time series.

Table 4: The results of the ablation experiment

| Metric | Models | Dataset | | | | | | | |
|--------|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | | Electricity | ETTh1 | ETTh2 | ETTm1 | ETTm2 | Exchange | Traffic | Weather |
| MSE | C+T | 0.215 | 0.441 | 0.380 | 0.403 | 0.286 | 0.362 | 0.623 | 0.260 |
| | C | 0.214 ↓ | 0.444 ↑ | 0.386 ↑ | 0.401 ↓ | 0.286 | 0.386 ↑ | 0.622 ↓ | 0.262 ↑ |
| | T | 0.215 | 0.445 ↑ | 0.380 | 0.414 ↑ | 0.286 | 0.363 ↑ | 0.623 | 0.273 ↑ |
| MAE | C+T | 0.292 | 0.429 | 0.403 | 0.398 | 0.327 | 0.402 | 0.372 | 0.283 |
| | C | 0.291 ↓ | 0.430 ↑ | 0.407 ↑ | 0.397 ↓ | 0.329 ↑ | 0.414 ↑ | 0.373 ↑ | 0.284 ↑ |
| | T | 0.292 | 0.432 ↑ | 0.402 ↓ | 0.408 ↑ | 0.328 ↑ | 0.402 | 0.372 | 0.291 ↑ |

↑: Larger Error than C+T ↓: Smaller Error than C+T

4.4 Efficiency Comparison

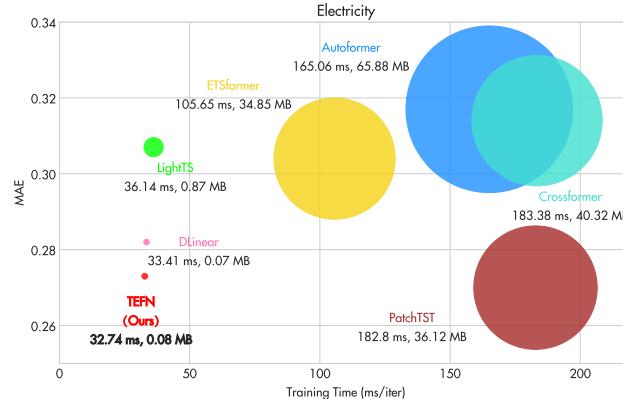


Figure 6: Model efficiency comparison. Results are from the prediction task Electricity-96, where the model size refers to the size of the binary model files, and iteration time is averaged each iterations.

TEFN is a compact model compared to the parameters of the Transformer. In order to effectively compare the efficiency of the model, we chose the dataset Electricity with a large dataset for performance comparison. In the task Electricity-96, we measured the average time, MSE, and exported model file size for iterating over one sample. Different models occupy varying amounts of GPU and memory. Therefore, we used the size of the saved binary model files as the model size. We visualized this with a bubble chart in Figure 6 . It can be seen that TEFN achieves a low prediction error with an extremely small number of parameters. Meanwhile, it has the shortest average time for iterating over one sample, demonstrating its high efficiency. TEFN maintains almost Dlinear [18] performance and almost PatchTST [12] accuracy, making it a performance error balanced model.

4.5 Interpretability Analysis

TEFN will allocate mass based on the event space $E = \{E_i | i \in [1, 2^{|S|}]\}$, and mass indicates the confidence reflecting support of different possible events. This support allocation is interpretable because BPA allocation utilizes fuzzy membership $\mu(x) = wx + b$ in Figure 7. Essentially, different fuzzy sets are constructed through TEFN for inference.

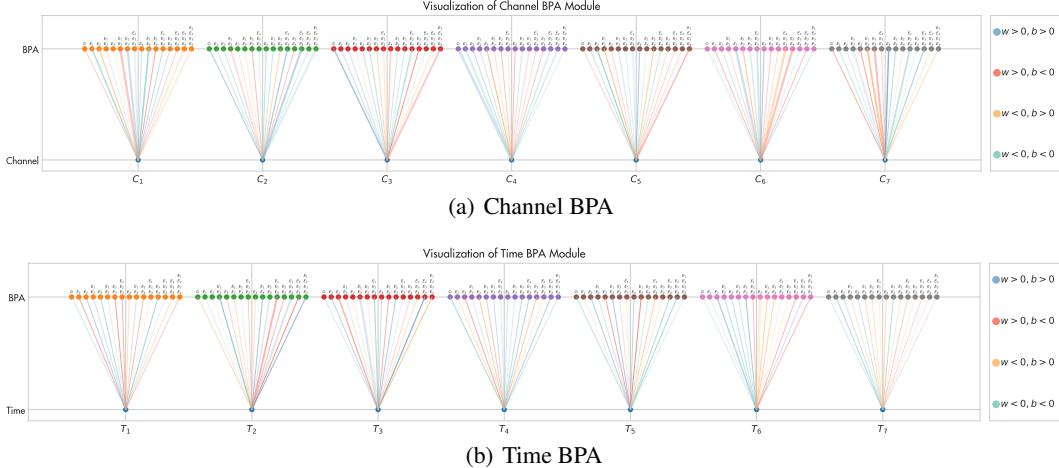


Figure 7: Visualization of Time BPA and Channel BPA modules in target ETTh1-96: Each line represents the fuzzy set membership function corresponding to the time. The fuzzy numbers established in TEFN are all triangular fuzzy numbers, and different channels C_i and time T_i can correspond to fuzzy sets that conform to symbolic logic.

4.6 Limitation and Future Prospects

TEFN is a naive and unmodified model. We lack skills in the application of BPA modules, such as how to generate BPA, and fuzzy logic is not the optimal implementation method. Due to fuzzy logic, TEFN is essentially a linear model that cannot handle some nonlinear time series well. In addition, for the fusion of mass distribution in the face of the failure of the classic Dempster Shafer rule for a large amount of data, using expectations as the fusion method may not necessarily be the optimal pattern. In terms of performance optimization, TEFN adopts a simple fully connected approach, which can lead to a rapid increase in parameter count for time series that are too long or have multiple channels. In fact, the implementation techniques of TEFN can further optimize the performance of the model through sampling and kernel operations similar to convolution. Thanks to the fact that evidence theory belongs to symbolic logic, the interpretation of neural network parameters will be more in line with human logic. Effective parameter initialization may exist, for example, initializing corresponding parameters in a non neural network way may lead to faster convergence.

5 Conclusion

This paper treats different channels and different time points of time series as multiple sources of information. By utilizing evidence theory to construct different basic probability assignments, TEFN discards irrelevant time points and channels. Experimentally, TEFN is a parameter-insensitive and stable prediction model with a small parameter count and fast training speed. In future work, we will explore how to optimize basic probability assignments and information fusion in neural networks.

Acknowledgments

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References

- [1] Electricity Load Time Series Dataset. <https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014/>.
- [2] Traffic Dataset. <http://pems.dot.ca.gov/>.
- [3] Weather Dataset. <https://www.bgc-jena.mpg.de/wetter/>.
- [4] Yoshua Bengio, Yann LeCun, and Donnie Henderson. Globally trained handwritten word recognizer using spatial representation, convolutional neural networks, and hidden markov models. *Advances in neural information processing systems*, 6, 1993.
- [5] Abhimanyu Das, Weihao Kong, Andrew Leach, Rajat Sen, and Rose Yu. Long-term forecasting with tide: Time-series dense encoder. *arXiv preprint arXiv:2304.08424*, 2023.
- [6] AP DEMPSTER. Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, 38:325–339, 1967.
- [7] Yong Deng. Generalized evidence theory. *Applied Intelligence*, 43(3):530–543, 2015.
- [8] Thierry Denoeux. A neural network classifier based on dempster-shafer theory. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 30(2):131–150, 2000.
- [9] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *ICLR*, 2015.
- [10] Jianxin Li, Xiong Hui, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. *arXiv: 2012.07436*, 2021.
- [11] Yong Liu, Haixu Wu, Jianmin Wang, and Mingsheng Long. Non-stationary transformers: Rethinking the stationarity in time series forecasting. *NeurIPS*, 2022.
- [12] Yuqi Nie, Nam H Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. A time series is worth 64 words: Long-term forecasting with transformers. *ICLR*, 2023.
- [13] Adam Paszke, S. Gross, Francisco Massa, A. Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Z. Lin, N. Gimelshein, L. Antiga, Alban Desmaison, Andreas Köpf, Edward Yang, Zach DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library. *NeurIPS*, 2019.
- [14] Glenn Shafer. *A mathematical theory of evidence*, volume 42. Princeton university press, 1976.
- [15] Gerald Woo, Chenghao Liu, Doyen Sahoo, Akshat Kumar, and Steven C. H. Hoi. Etsformer: Exponential smoothing transformers for time-series forecasting. *arXiv preprint arXiv:2202.01381*, 2022.
- [16] Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. *ICLR*, 2023.
- [17] Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with Auto-Correlation for long-term series forecasting. *NeurIPS*, 2021.
- [18] Ailing Zeng, Muxi Chen, Lei Zhang, and Qiang Xu. Are transformers effective for time series forecasting? *AAAI*, 2023.
- [19] Tianping Zhang, Yizhuo Zhang, Wei Cao, Jiang Bian, Xiaohan Yi, Shun Zheng, and Jian Li. Less is more: Fast multivariate time series forecasting with light sampling-oriented mlp structures. *arXiv preprint arXiv:2207.01186*, 2022.
- [20] Yunhao Zhang and Junchi Yan. Crossformer: Transformer utilizing cross-dimension dependency for multivariate time series forecasting. *ICLR*, 2023.
- [21] Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. FEDformer: Frequency enhanced decomposed transformer for long-term series forecasting. *ICML*, 2022.

A Implementation Details

Due to limited experimental equipment, our experimental environment was a regular desktop computer with a single CPU and a single GPU without paralleled computing of multi-GPU. Our experimental equipment is configured with AMD Ryzen9 7950x 4.5GHz 16 Cores , 64GB RAM DDR5 4800MHz, and NVIDIA RTX 4090 D 24GB. The proposed model TEFN is implemented in the PyTorch [13]. For the sake of fairness, we used an open-source benchmark located at <https://github.com/thuml/Time-Series-Library> which contains comparison models and corresponding experiment configurations. The loss function selected during the experiment was MSE, and the optimizer selected Adam [9]. We used Mean Absolute Error (MAE) and Mean Square Error (MSE) in Equation 9 and Equation 10 as error indicators in the experiment where y_i is actual value and \hat{y}_i is predicted value.

$$MAE(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (9)$$

$$MSE(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (10)$$

B Supplementary Experimental Results

B.1 Dataset Feature

In the experimental section, we mainly referred to the experimental dataset and experimental settings of TimesNet. The experimental settings of TimesNet are comprehensive and reasonable [16]. Table 5 shows the quantitative features of the dataset used in the experiment. Dataset size corresponds to the training set, validation set, and test set. Each dataset has four forecasting tasks of different lengths $L \in \{96, 192, 336, 720\}$.

Table 5: The quantitative features of the dataset

| Dataset | Dimension | Dataset Size |
|--------------|-----------|-----------------------|
| Electricity | 321 | (18317, 2633, 5261) |
| ETTm1, ETTm2 | 7 | (34465, 11521, 11521) |
| ETTh1, ETTh2 | 7 | (8545, 2881, 2881) |
| Traffic | 862 | (12185, 1757, 3509) |
| Weather | 21 | (36792, 5271, 10540) |
| Exchange | 8 | (5120, 665, 1422) |

B.2 Forecasting Visualization

We visualized the predictions of TEFN and comparative models to better demonstrate the predictive performance of different models. We take the task of Electricity-96 [1] as an example and selected typical models PatchTST [12], Crossformer [20], TimesNet [16], Dlinear [18] and Stationary [11] for visualization in Figure 8. From the visualization results, TEFN fits the details and trends of time series better.

B.3 Full Comparative Results

In the comparative experiment, we conducted a thorough comparison of 5 datasets and 10 comparative models in terms of forecasting length $l \in \{96, 192, 336, 720\}$ in Table 7. It can be seen that TEFN has positive predictive performance on the experiment dataset, achieving SOTA in multiple forecasting tasks across multiple datasets.

B.4 Full Ablation Study

We conducted ablation experiments on all forecasting tasks, testing individual channel C forecasting and individual time T forecasting. We tested the forecasting length $L \in \{96, 192, 336, 720\}$ and averaged Avg the errors of the four forecasting lengths in Table 6. In the aspect of all the results of ablation, TEFN can effectively improve the performance by combining the dimensions of time T and channel C in most cases. There are a few cases where the effect is not as good as the individual dimensions, but the difference is small and can be ignored.

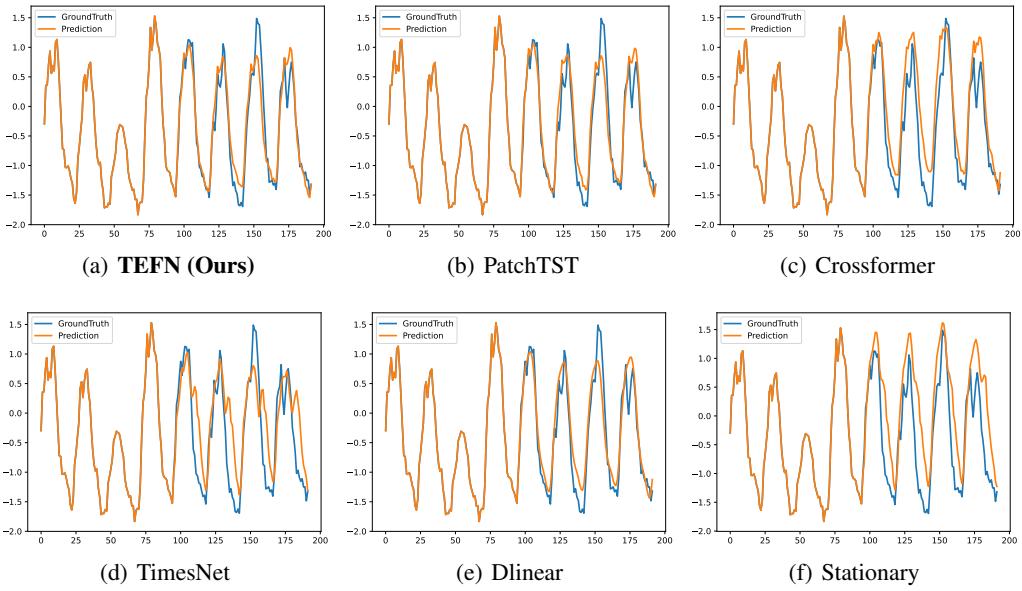


Figure 8: Visualization of Electricity-96 tasks

Table 6: Full results of the ablation experiment

| Metric | Target | Model | Dataset | | | | | | | | | | | |
|--------|--------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-------|--|----------|---------|
| | | | Electricity | | ETTh1 | | ETTh2 | | ETTm1 | | ETTm2 | | Exchange | Traffic |
| MSE | 96 | C+T | 0.197 | 0.383 | 0.288 | 0.343 | 0.181 | 0.082 | 0.645 | 0.182 | | | | |
| | | C | 0.196 ↓ | 0.386 ↑ | 0.288 | 0.339 ↓ | 0.185 ↑ | 0.083 ↑ | 0.644 ↓ | 0.185 ↑ | | | | |
| | | T | 0.197 | 0.386 ↑ | 0.291 ↑ | 0.356 ↑ | 0.182 ↑ | 0.083 ↑ | 0.645 | 0.195 ↑ | | | | |
| | 192 | C+T | 0.197 | 0.433 | 0.375 | 0.381 | 0.246 | 0.176 | 0.598 | 0.227 | | | | |
| | | C | 0.196 ↓ | 0.435 ↑ | 0.375 | 0.382 ↑ | | 0.179 ↑ | 0.597 ↓ | 0.230 ↑ | | | | |
| | | T | 0.197 | 0.435 ↑ | 0.375 | 0.390 ↑ | 0.247 ↑ | 0.175 ↓ | 0.598 | 0.240 ↑ | | | | |
| | 336 | C+T | 0.212 | 0.475 | 0.423 | 0.414 | 0.307 | 0.328 | 0.605 | 0.279 | | | | |
| | | C | 0.210 ↓ | 0.479 ↑ | 0.414 ↓ | 0.411 ↓ | 0.307 | 0.336 ↑ | 0.604 ↓ | 0.280 ↑ | | | | |
| | | T | 0.212 | 0.479 ↑ | 0.414 ↓ | 0.423 ↑ | 0.307 | 0.324 ↓ | 0.606 ↑ | 0.291 ↑ | | | | |
| | 720 | C+T | 0.253 | 0.475 | 0.434 | 0.475 | 0.407 | 0.861 | 0.644 | 0.352 | | | | |
| | | C | 0.253 | 0.478 ↑ | 0.457 ↑ | 0.472 ↓ | 0.407 | 0.947 ↑ | 0.642 ↓ | 0.354 ↑ | | | | |
| | | T | 0.254 ↑ | 0.479 ↑ | 0.442 ↑ | 0.487 ↑ | 0.408 ↑ | 0.871 ↑ | 0.643 ↓ | 0.364 ↑ | | | | |
| | Avg | C+T | 0.215 | 0.441 | 0.380 | 0.403 | 0.286 | 0.362 | 0.623 | 0.260 | | | | |
| | | C | 0.214 ↓ | 0.444 ↑ | 0.386 ↑ | 0.401 ↓ | 0.286 | 0.386 ↑ | 0.622 ↓ | 0.262 ↑ | | | | |
| | | T | 0.215 | 0.445 ↑ | 0.380 | 0.414 ↑ | 0.286 | 0.363 ↑ | 0.623 | 0.273 ↑ | | | | |
| | 96 | C+T | 0.274 | 0.391 | 0.338 | 0.367 | 0.264 | 0.198 | 0.384 | 0.227 | | | | |
| | | C | 0.273 ↓ | 0.394 ↑ | 0.339 ↑ | 0.366 ↓ | 0.269 ↑ | 0.199 ↑ | 0.384 | 0.228 ↑ | | | | |
| | | T | 0.274 | 0.393 ↑ | 0.339 ↑ | 0.377 ↑ | 0.265 ↑ | 0.199 ↑ | 0.383 ↓ | 0.234 ↑ | | | | |
| | 192 | C+T | 0.276 | 0.420 | 0.392 | 0.384 | 0.304 | 0.297 | 0.360 | 0.262 | | | | |
| | | C | 0.275 ↓ | 0.421 ↑ | 0.393 ↑ | 0.385 ↑ | 0.305 ↑ | 0.300 ↑ | 0.361 ↑ | 0.264 ↑ | | | | |
| | | T | 0.276 | 0.421 ↑ | 0.390 ↓ | 0.392 ↑ | 0.305 ↑ | 0.296 ↓ | 0.361 ↑ | 0.270 ↑ | | | | |
| | 336 | C+T | 0.292 | 0.441 | 0.435 | 0.405 | 0.343 | 0.412 | 0.363 | 0.298 | | | | |
| | | C | 0.291 ↓ | 0.443 ↑ | 0.434 ↓ | 0.403 ↓ | 0.343 | 0.419 ↑ | 0.363 | 0.298 | | | | |
| | | T | 0.292 | 0.445 ↑ | 0.425 ↓ | 0.413 ↑ | 0.342 ↓ | 0.410 ↓ | 0.362 ↓ | 0.306 ↑ | | | | |
| | 720 | C+T | 0.325 | 0.465 | 0.446 | 0.438 | 0.399 | 0.700 | 0.382 | 0.344 | | | | |
| | | C | 0.325 | 0.463 ↓ | 0.462 ↑ | 0.436 ↓ | 0.399 | 0.739 ↑ | 0.382 | 0.347 ↑ | | | | |
| | | T | 0.324 ↓ | 0.469 ↑ | 0.454 ↑ | 0.449 ↑ | 0.398 ↓ | 0.702 ↑ | 0.382 | 0.354 ↑ | | | | |
| | Avg | C+T | 0.292 | 0.429 | 0.403 | 0.398 | 0.327 | 0.402 | 0.372 | 0.283 | | | | |
| | | C | 0.291 ↓ | 0.430 ↑ | 0.407 ↑ | 0.397 ↓ | 0.329 ↑ | 0.414 ↑ | 0.373 ↑ | 0.284 ↑ | | | | |
| | | T | 0.292 | 0.432 ↑ | 0.402 ↓ | 0.408 ↑ | 0.328 ↑ | 0.402 | 0.372 | 0.291 ↑ | | | | |

↑: Larger Error than C+T ↓: Smaller Error than C+T

Table 7: Full results of comparative results

| Models | TEFN (Ours) | PatchTST [12] | Crossformer [20] | TIDE [5] | TimesNet [16] | ETsformer [15] | LightTS [19] | Dlinear [18] | FEDformer [21] | Stationary [11] | Autoformer [17] | | | | | | | | | |
|---|----------------|--|--|----------------|---------------------------|------------------------------------|------------------------------------|------------------------------------|---------------------------|-----------------------|------------------------------------|--------------|------------------------------------|--------------|---------------------------|-------|---------------------------|-------|-------|-------|
| Metric | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | | | | | | | | |
| ETTm1 | 96 | 0.343 0.367 0.329 0.367 | 0.404 | 0.426 | 0.364 | 0.387 0.338 0.375 | 0.375 | 0.398 | 0.374 | 0.400 | 0.345 0.372 | 0.379 | 0.419 | 0.386 | 0.398 | 0.505 | 0.475 | | | |
| | 192 | 0.381 | 0.383 0.367 0.385 | 0.450 | 0.451 | 0.398 | 0.404 0.374 0.387 | 0.408 | 0.410 | 0.400 | 0.407 0.380 | 0.389 | 0.426 | 0.441 | 0.459 | 0.444 | 0.553 | 0.496 | | |
| | 336 | 0.414 | 0.404 0.399 0.410 | 0.532 | 0.515 | 0.428 | 0.425 0.410 0.411 | 0.435 | 0.428 | 0.438 | 0.438 0.413 | 0.413 | 0.445 | 0.459 | 0.495 | 0.464 | 0.621 | 0.537 | | |
| | 720 | 0.475 0.438 0.454 0.439 | 0.666 | 0.589 | 0.487 | 0.461 0.478 0.450 | 0.499 | 0.462 | 0.527 | 0.502 0.474 | 0.453 | 0.543 | 0.490 | 0.585 | 0.516 | 0.671 | 0.561 | | | |
| Avg 0.403 0.398 0.387 0.400 | | | | | | | | | | | | | | | | | | | | |
| ETTm2 | 96 | 0.181 0.264 0.175 0.259 | 0.287 | 0.366 | 0.207 | 0.305 0.187 0.267 | 0.189 | 0.280 | 0.209 | 0.308 0.193 | 0.292 | 0.203 | 0.287 | 0.192 | 0.274 | 0.255 | 0.339 | | | |
| | 192 | 0.246 0.304 0.241 0.302 | 0.414 | 0.492 | 0.290 | 0.364 0.249 0.309 | 0.253 | 0.319 | 0.311 | 0.382 0.284 | 0.362 | 0.269 | 0.328 | 0.280 | 0.339 | 0.281 | 0.340 | | | |
| | 336 | 0.307 0.343 0.305 0.343 | 0.597 | 0.542 | 0.377 | 0.422 0.321 | 0.351 0.314 | 0.357 | 0.442 | 0.466 0.369 | 0.427 | 0.325 | 0.366 | 0.334 | 0.361 | 0.339 | 0.372 | | | |
| | 720 | 0.407 0.398 0.402 0.400 | 1.730 | 1.042 | 0.558 | 0.524 0.408 0.403 | 0.414 | 0.413 | 0.675 | 0.587 0.554 | 0.522 | 0.421 | 0.415 | 0.417 | 0.413 | 0.433 | 0.432 | | | |
| Avg 0.286 0.327 0.281 0.326 | | | | | | | | | | | | | | | | | | | | |
| ETTh1 | 96 | 0.383 0.391 0.384 0.392 | 0.414 | 0.419 | 0.423 | 0.448 | 0.479 | 0.464 0.384 0.402 | 0.494 | 0.479 | 0.424 | 0.432 | 0.386 0.400 0.376 | 0.419 | 0.513 | 0.491 | 0.449 | 0.459 | | |
| | 192 | 0.433 0.419 0.460 0.445 | 0.471 | 0.474 | 0.525 | 0.492 0.436 0.429 | 0.538 | 0.504 | 0.475 | 0.462 | 0.437 0.432 0.420 | 0.448 | 0.534 | 0.504 | 0.500 | 0.482 | | | | |
| | 336 | 0.475 0.441 0.501 0.466 | 0.570 | 0.546 | 0.565 | 0.515 0.491 | 0.469 | 0.574 | 0.521 | 0.518 | 0.488 0.481 0.459 | 0.459 | 0.588 | 0.535 | 0.521 | 0.496 | | | | |
| | 720 | 0.475 0.464 0.500 0.488 | 0.653 | 0.621 | 0.594 | 0.558 0.521 | 0.500 | 0.562 | 0.535 | 0.547 | 0.533 0.519 | 0.516 | 0.506 | 0.507 | 0.643 | 0.616 | 0.514 | 0.512 | | |
| Avg 0.441 0.429 0.469 0.454 | | | | | | | | | | | | | | | | | | | | |
| ETTh2 | 96 | 0.288 0.337 0.302 0.348 | 0.745 | 0.584 | 0.400 | 0.440 0.340 | 0.374 | 0.340 | 0.391 | 0.397 | 0.437 0.333 0.387 | 0.358 | 0.397 | 0.476 | 0.458 | 0.346 | 0.388 | | | |
| | 192 | 0.375 0.392 0.388 0.400 | 0.877 | 0.656 | 0.528 | 0.509 0.402 0.414 | 0.430 | 0.439 | 0.520 | 0.504 | 0.477 | 0.476 | 0.429 | 0.439 | 0.512 | 0.493 | 0.456 | 0.452 | | |
| | 336 | 0.423 0.434 0.426 0.433 | 1.043 | 0.731 | 0.643 | 0.571 0.452 0.452 | 0.485 | 0.479 | 0.626 | 0.559 | 0.594 | 0.541 | 0.496 | 0.487 | 0.552 | 0.511 | 0.482 | 0.486 | | |
| | 720 | 0.434 0.446 0.431 0.446 | 1.104 | 0.763 | 0.874 | 0.679 0.462 0.468 | 0.500 | 0.497 | 0.863 | 0.672 | 0.831 | 0.657 | 0.463 | 0.474 | 0.562 | 0.560 | 0.515 | 0.511 | | |
| Avg 0.380 0.402 0.387 0.407 | | | | | | | | | | | | | | | | | | | | |
| Electricity | 96 | 0.197 0.273 | 0.181 0.270 | 0.219 | 0.314 | 0.237 | 0.329 0.168 0.272 | 0.187 | 0.304 | 0.207 | 0.307 0.197 | 0.282 | 0.193 | 0.308 | 0.169 | 0.273 | 0.201 | 0.317 | | |
| | 192 | 0.197 0.270 | 0.188 0.274 | 0.231 | 0.322 | 0.230 | 0.330 0.184 0.289 | 0.199 | 0.315 | 0.213 | 0.316 0.196 | 0.285 | 0.201 | 0.315 | 0.182 | 0.286 | 0.222 | 0.334 | | |
| | 336 | 0.212 0.292 | 0.204 0.293 | 0.246 | 0.337 | 0.249 | 0.344 0.198 0.300 | 0.212 | 0.329 | 0.230 | 0.333 0.209 | 0.301 | 0.214 | 0.329 | 0.200 | 0.304 | 0.231 | 0.338 | | |
| | 720 | 0.253 0.325 | 0.246 0.324 | 0.280 | 0.363 | 0.284 | 0.373 0.220 0.320 | 0.233 | 0.345 | 0.265 | 0.360 0.245 | 0.333 | 0.246 | 0.355 | 0.222 0.321 | 0.254 | 0.361 | | | |
| Avg 0.215 0.292 0.205 0.290 | | | | | | | | | | | | | | | | | | | | |
| Exchange | 96 | 0.082 0.198 0.177 0.218 | 0.158 | 0.230 | 0.202 | 0.261 0.107 | 0.234 | 0.085 0.204 | 0.116 | 0.262 | 0.088 0.218 | 0.148 | 0.278 | 0.111 | 0.237 | 0.197 | 0.323 | | | |
| | 192 | 0.176 0.297 0.225 0.259 | 0.206 | 0.277 | 0.242 | 0.298 | 0.226 | 0.182 0.303 | 0.215 | 0.359 | 0.176 0.315 | 0.271 | 0.380 | 0.219 | 0.335 | 0.300 | 0.369 | | | |
| | 336 | 0.328 0.412 | 0.278 0.297 0.272 0.335 | 0.377 0.311 | 0.371 0.335 | 0.349 0.337 | 0.337 0.287 0.335 | 0.367 | 0.448 | 0.348 | 0.428 0.377 | 0.466 | 0.313 | 0.427 | 0.460 | 0.500 | 0.421 | 0.476 | 0.509 | 0.524 |
| | 720 | 0.861 0.700 | 0.354 0.348 0.398 0.418 | 0.351 0.386 | 0.351 0.386 | 0.964 0.746 | 0.746 1.025 | 0.774 0.774 | 0.831 | 0.699 | 0.839 0.695 | 0.695 | 1.195 | 0.841 | 1.092 | 0.769 | 1.447 | 0.941 | | |
| Avg 0.362 0.372 0.367 0.404 | | | | | | | | | | | | | | | | | | | | |
| Weather | 96 | 0.182 0.227 0.162 0.262 | 0.462 | 0.295 | 0.522 | 0.290 | 0.805 | 0.493 0.172 0.220 | 0.197 | 0.281 | 0.182 | 0.242 | 0.196 | 0.255 | 0.217 | 0.296 | 0.173 0.223 | 0.266 | 0.336 | |
| | 192 | 0.227 0.262 0.246 0.296 | 0.530 | 0.293 | 0.756 | 0.474 0.219 0.261 | 0.237 | 0.312 | 0.227 0.287 | 0.237 | 0.296 | 0.276 | 0.336 | 0.245 | 0.285 | 0.307 | 0.367 | | | |
| | 336 | 0.279 0.298 0.482 0.304 | 0.558 | 0.305 | 0.762 | 0.477 0.280 | 0.306 | 0.298 | 0.282 0.334 | 0.283 | 0.335 | 0.339 | 0.380 | 0.321 | 0.338 | 0.359 | 0.395 | | | |
| | 720 | 0.352 0.344 0.514 0.322 | 0.589 | 0.328 | 0.719 | 0.449 0.365 | 0.359 | 0.352 0.288 | 0.352 | 0.386 | 0.345 0.381 | 0.403 | 0.428 | 0.414 | 0.410 | 0.419 | 0.428 | | | |
| Avg 0.260 0.283 0.481 0.304 | | | | | | | | | | | | | | | | | | | | |
| 1st Count 8 20 16 13 1 0 1 0 7 5 0 1 0 0 1 0 5 0 1 1 1 0 0 | | | | | | | | | | | | | | | | | | | | |
| 2nd Count 13 7 7 15 1 3 4 3 6 5 2 1 0 0 2 2 0 0 0 5 4 1 0 0 | | | | | | | | | | | | | | | | | | | | |
| 3rd Count 5 5 4 3 3 3 2 2 11 20 4 0 3 0 6 4 2 1 0 1 1 0 1 | | | | | | | | | | | | | | | | | | | | |

According to the characteristics of neural networks, the forecasting error of TEFN l_{C+T} will be less than or equal to using separate time l_T or channel l_C in Equation 11.

$$l_{C+T} \leqslant \min(l_C, l_T) \quad (11)$$

B.5 Full Hyperparameter Sensitivity

The GPU memory in the experimental environment is limited, so we conducted a 2-dimensional traversal of the sample space size $|S| \in \{0, 1, 2, 3, 4, 5, 6\}$ and learning rate $lr \in \{1 * 10^{-2}, 5 * 10^{-2}, 1 * 10^{-1}\}$ in Table 8. In the case of sufficient GPU memory, more hyperparameters can be traversed. Based on the complete results of hyperparameter sensitivity, it can be seen that TEFN is an insensitive model to hyperparameters, which means that random hyperparameters can achieve stable and superior prediction performance. We use variance to describe the fluctuation of hyperparameters in Table 3.

Table 8: Full results of hyperparameter sensitivity

| S | lr | Target | Dataset | | | | | | | | | | | | | | | | | | | | | |
|---|---------------|--------|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|-----|---------|--|---------|--|
| | | | Electricity | | | | ETTh1 | | | | ETTh2 | | | | ETTm1 | | ETTm2 | | Exchange | | Traffic | | Weather | |
| | | | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | | | | |
| 0 | $1 * 10^{-2}$ | 96 | 0.197 | 0.273 | 0.403 | 0.407 | 0.288 | 0.337 | 0.344 | 0.367 | 0.182 | 0.264 | 0.086 | 0.204 | 0.645 | 0.384 | 0.186 | 0.230 | | | | | | |
| | | 192 | 0.197 | 0.276 | 0.433 | 0.420 | 0.375 | 0.392 | 0.385 | 0.385 | 0.247 | 0.305 | 0.179 | 0.300 | 0.598 | 0.360 | 0.232 | 0.266 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.478 | 0.443 | 0.423 | 0.434 | 0.414 | 0.404 | 0.310 | 0.344 | 0.328 | 0.412 | 0.605 | 0.362 | 0.283 | 0.301 | | | | | | |
| | | 720 | 0.254 | 0.326 | 0.476 | 0.465 | 0.452 | 0.458 | 0.478 | 0.438 | 0.410 | 0.400 | 0.886 | 0.711 | 0.644 | 0.382 | 0.357 | 0.348 | | | | | | |
| | $5 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.383 | 0.391 | 0.291 | 0.340 | 0.347 | 0.369 | 0.182 | 0.265 | 0.082 | 0.198 | 0.645 | 0.384 | 0.182 | 0.227 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.433 | 0.420 | 0.376 | 0.393 | 0.383 | 0.385 | 0.247 | 0.305 | 0.179 | 0.300 | 0.598 | 0.360 | 0.232 | 0.266 | | | | | | |
| | | 336 | 0.213 | 0.294 | 0.476 | 0.441 | 0.425 | 0.437 | 0.415 | 0.406 | 0.308 | 0.344 | 0.340 | 0.421 | 0.605 | 0.363 | 0.284 | 0.301 | | | | | | |
| | | 720 | 0.254 | 0.327 | 0.478 | 0.465 | 0.453 | 0.459 | 0.478 | 0.439 | 0.408 | 0.399 | 0.911 | 0.722 | 0.644 | 0.382 | 0.352 | 0.344 | | | | | | |
| | $1 * 10^{-1}$ | 96 | 0.197 | 0.274 | 0.383 | 0.391 | 0.291 | 0.340 | 0.383 | 0.385 | 0.182 | 0.264 | 0.082 | 0.198 | 0.645 | 0.383 | 0.183 | 0.228 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.433 | 0.419 | 0.376 | 0.394 | 0.257 | 0.314 | 0.180 | 0.300 | 0.598 | 0.360 | 0.244 | 0.274 | | | | | | | | |
| | | 336 | 0.214 | 0.295 | 0.476 | 0.441 | 0.424 | 0.435 | 0.416 | 0.406 | 0.309 | 0.344 | 0.340 | 0.421 | 0.605 | 0.363 | 0.292 | 0.308 | | | | | | |
| | | 720 | 0.255 | 0.327 | 0.482 | 0.469 | 0.454 | 0.460 | 0.477 | 0.439 | 0.408 | 0.400 | 0.937 | 0.732 | 0.644 | 0.382 | 0.352 | 0.345 | | | | | | |
| 1 | $1 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.384 | 0.392 | 0.289 | 0.337 | 0.349 | 0.370 | 0.182 | 0.265 | 0.083 | 0.199 | 0.645 | 0.385 | 0.186 | 0.229 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.438 | 0.421 | 0.377 | 0.392 | 0.382 | 0.386 | 0.246 | 0.304 | 0.178 | 0.299 | 0.598 | 0.361 | 0.232 | 0.265 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.475 | 0.441 | 0.425 | 0.435 | 0.418 | 0.407 | 0.307 | 0.343 | 0.335 | 0.418 | 0.606 | 0.363 | 0.280 | 0.400 | | | | | | |
| | | 720 | 0.253 | 0.325 | 0.476 | 0.464 | 0.452 | 0.458 | 0.482 | 0.442 | 0.409 | 0.400 | 0.883 | 0.709 | 0.644 | 0.382 | 0.355 | 0.347 | | | | | | |
| | $5 * 10^{-2}$ | 96 | 0.198 | 0.275 | 0.386 | 0.393 | 0.294 | 0.341 | 0.346 | 0.369 | 0.182 | 0.265 | 0.088 | 0.205 | 0.645 | 0.385 | 0.194 | 0.239 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.438 | 0.422 | 0.375 | 0.392 | 0.383 | 0.385 | 0.247 | 0.305 | 0.184 | 0.304 | 0.598 | 0.360 | 0.241 | 0.274 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.476 | 0.441 | 0.423 | 0.435 | 0.415 | 0.404 | 0.308 | 0.343 | 0.361 | 0.434 | 0.605 | 0.363 | 0.282 | 0.300 | | | | | | |
| | | 720 | 0.256 | 0.328 | 0.476 | 0.466 | 0.463 | 0.465 | 0.478 | 0.439 | 0.408 | 0.398 | 0.892 | 0.710 | 0.644 | 0.383 | 0.360 | 0.353 | | | | | | |
| | $1 * 10^{-1}$ | 96 | 0.197 | 0.274 | 0.383 | 0.391 | 0.290 | 0.338 | 0.347 | 0.369 | 0.183 | 0.266 | 0.083 | 0.200 | 0.645 | 0.383 | 0.187 | 0.230 | | | | | | |
| | | 192 | 0.197 | 0.276 | 0.434 | 0.420 | 0.375 | 0.393 | 0.386 | 0.386 | 0.247 | 0.305 | 0.177 | 0.298 | 0.598 | 0.360 | 0.232 | 0.266 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.476 | 0.441 | 0.423 | 0.435 | 0.414 | 0.405 | 0.311 | 0.346 | 0.348 | 0.427 | 0.605 | 0.362 | 0.284 | 0.301 | | | | | | |
| | | 720 | 0.253 | 0.325 | 0.476 | 0.466 | 0.453 | 0.459 | 0.478 | 0.439 | 0.408 | 0.399 | 0.898 | 0.719 | 0.644 | 0.382 | 0.355 | 0.347 | | | | | | |
| 2 | $5 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.383 | 0.391 | 0.289 | 0.338 | 0.345 | 0.368 | 0.182 | 0.265 | 0.084 | 0.202 | 0.646 | 0.384 | 0.183 | 0.228 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.434 | 0.420 | 0.377 | 0.394 | 0.384 | 0.387 | 0.247 | 0.305 | 0.177 | 0.298 | 0.598 | 0.360 | 0.228 | 0.263 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.477 | 0.443 | 0.424 | 0.436 | 0.415 | 0.405 | 0.308 | 0.343 | 0.353 | 0.430 | 0.605 | 0.363 | 0.293 | 0.309 | | | | | | |
| | | 720 | 0.254 | 0.325 | 0.478 | 0.466 | 0.458 | 0.461 | 0.477 | 0.439 | 0.408 | 0.400 | 0.930 | 0.732 | 0.644 | 0.382 | 0.355 | 0.347 | | | | | | |
| | $1 * 10^{-1}$ | 96 | 0.197 | 0.274 | 0.384 | 0.391 | 0.289 | 0.339 | 0.347 | 0.368 | 0.182 | 0.265 | 0.088 | 0.205 | 0.646 | 0.385 | 0.186 | 0.231 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.438 | 0.423 | 0.378 | 0.395 | 0.387 | 0.387 | 0.247 | 0.305 | 0.191 | 0.310 | 0.598 | 0.361 | 0.241 | 0.278 | | | | | | |
| | | 336 | 0.212 | 0.293 | 0.480 | 0.446 | 0.425 | 0.437 | 0.416 | 0.406 | 0.308 | 0.343 | 0.343 | 0.463 | 0.605 | 0.363 | 0.289 | 0.316 | | | | | | |
| | | 720 | 0.254 | 0.326 | 0.476 | 0.466 | 0.463 | 0.465 | 0.478 | 0.439 | 0.410 | 0.402 | 0.880 | 0.707 | 0.644 | 0.382 | 0.357 | 0.350 | | | | | | |
| | $1 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.385 | 0.393 | 0.290 | 0.339 | 0.343 | 0.368 | 0.182 | 0.265 | 0.083 | 0.199 | 0.645 | 0.383 | 0.183 | 0.227 | | | | | | |
| | | 192 | 0.197 | 0.277 | 0.434 | 0.420 | 0.376 | 0.393 | 0.383 | 0.385 | 0.248 | 0.306 | 0.183 | 0.304 | 0.598 | 0.360 | 0.232 | 0.266 | | | | | | |
| | | 336 | 0.212 | 0.293 | 0.477 | 0.443 | 0.426 | 0.435 | 0.414 | 0.405 | 0.308 | 0.344 | 0.338 | 0.420 | 0.606 | 0.363 | 0.282 | 0.300 | | | | | | |
| | | 720 | 0.253 | 0.325 | 0.477 | 0.465 | 0.457 | 0.473 | 0.476 | 0.438 | 0.407 | 0.399 | 0.943 | 0.739 | 0.644 | 0.382 | 0.353 | 0.346 | | | | | | |
| 3 | $5 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.383 | 0.391 | 0.290 | 0.337 | 0.344 | 0.368 | 0.183 | 0.266 | 0.083 | 0.200 | 0.645 | 0.383 | 0.189 | 0.236 | | | | | | |
| | | 192 | 0.198 | 0.277 | 0.435 | 0.421 | 0.382 | 0.398 | 0.384 | 0.385 | 0.258 | 0.316 | 0.200 | 0.317 | 0.598 | 0.360 | 0.234 | 0.269 | | | | | | |
| | | 336 | 0.213 | 0.293 | 0.476 | 0.441 | 0.426 | 0.437 | 0.415 | 0.406 | 0.308 | 0.344 | 0.353 | 0.429 | 0.606 | 0.363 | 0.285 | 0.303 | | | | | | |
| | | 720 | 0.254 | 0.325 | 0.489 | 0.474 | 0.460 | 0.463 | 0.478 | 0.449 | 0.412 | 0.402 | 0.101 | 0.775 | 0.644 | 0.383 | 0.357 | 0.350 | | | | | | |
| | $1 * 10^{-1}$ | 96 | 0.197 | 0.274 | 0.384 | 0.391 | 0.291 | 0.339 | 0.346 | 0.369 | 0.184 | 0.267 | 0.084 | 0.201 | 0.645 | 0.384 | 0.196 | 0.245 | | | | | | |
| | | 192 | 0.198 | 0.277 | 0.437 | 0.423 | 0.381 | 0.398 | 0.385 | 0.386 | 0.248 | 0.306 | 0.176 | 0.297 | 0.598 | 0.360 | 0.254 | 0.290 | | | | | | |
| | | 336 | 0.212 | 0.292 | 0.475 | 0.441 | 0.430 | 0.440 | 0.416 | 0.406 | 0.328 | 0.361 | 0.362 | 0.435 | 0.607 | 0.363 | 0.288 | 0.308 | | | | | | |
| | | 720 | 0.254 | 0.326 | 0.477 | 0.465 | 0.457 | 0.462 | 0.477 | 0.439 | 0.409 | 0.399 | 0.889 | 0.713 | 0.644 | 0.383 | 0.353 | 0.345 | | | | | | |
| | $1 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.393 | 0.399 | 0.288 | 0.338 | 0.344 | 0.368 | 0.185 | 0.268 | 0.084 | 0.201 | 0.645 | 0.383 | 0.182 | 0.227 | | | | | | |
| | | 192 | 0.197 | 0.276 | 0.434 | 0.420 | 0.376 | 0.393 | 0.381 | 0.385 | 0.247 | 0.305 | 0.179 | 0.300 | 0.598 | 0.360 | 0.227 | 0.262 | | | | | | |
| | | 336 | 0.215 | 0.295 | 0.489 | 0.452 | 0.473 | 0.467 | 0.420 | 0.407 | 0.309 | 0.344 | 0.377 | 0.447 | 0.606 | 0.363 | 0.255 | 0.324 | | | | | | |
| | | 720 | 0.254 | 0.328 | 0.485 | 0.470 | 0.454 | 0.460 | 0.480 | 0.441 | 0.414 | 0.404 | 0.923 | 0.727 | 0.644 | 0.383 | 0.375 | 0.364 | | | | | | |
| 4 | $5 * 10^{-2}$ | 96 | 0.197 | 0.274 | 0.383 | 0.392 | 0.292 | 0.340 | 0.345 | 0.368 | 0.181 | 0.264 | 0.085 | 0.203 | 0.645 | 0.384 | 0.203 | 0.254 | | | | | | |
| | | 192 | 0.198 | 0.278 | 0.433 | 0.420 | 0.377 | 0.394 | 0.385 | 0.386 | 0.248 | 0.307 | 0.179 | 0.300 | 0.598 | 0.360 | 0.234 | 0.270 | | | | | | |
| | | 336 | 0.213 | 0.293 | 0.486 | 0.449 | 0.425 | 0.435 | 0.417 | 0.407 | 0.312 | 0.347 | 0.346 | 0.427 | 0.607 | 0.363 | 0.294 | 0.314 | | | | | | |
| | | 720 | 0.257 | 0.330 | 0.480 | 0.467 | 0.462 | 0.479 | 0.440 | 0.439 | 0.408 | 0.399 | 0.861 | 0.700 | 0.644 | 0.383 | 0.359 | 0.353 | | | | | | |
| | $1 * 10^{-1}$ | 96 | 0.198 | 0.274 | 0.384 | 0.392 | 0.293 | 0.342 | 0.358 | 0.376 | 0.183 | 0.266 | 0.085 | 0.203 | 0.646 | 0.384 | | | | | | | | |